Lunar investigations at the Kazan University: the physical libration – analytical and numerical approach, the lunar coordinate systems

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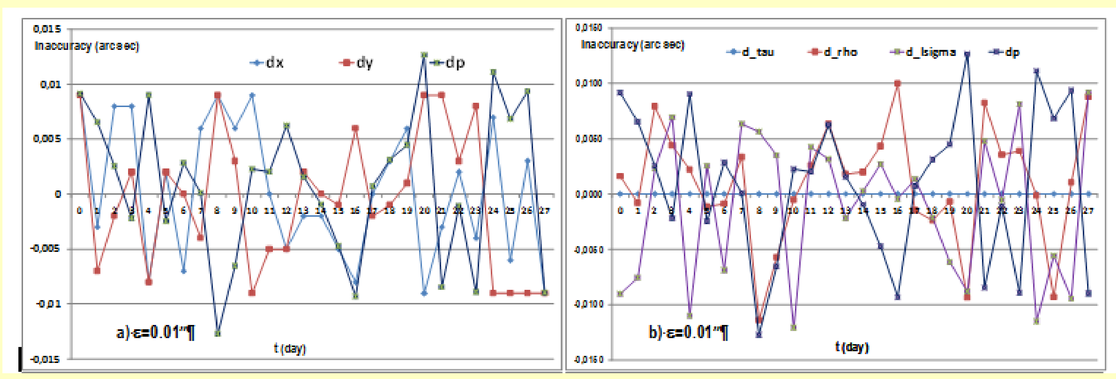
1. The theory of physical librations is one of traditional field of investigation at the Kazan University. At the present time it is necessary to develop the model of lunar rotation in order to achieve in the theory the accuracy of 0.1 milliseconds of arc, which is the requirement of modern laser ranging observations and other experiments to determine the parameters of the physical libration.
Both numerical and analytical approaches are very important, since the first provides greater accuracy, and the second - allows a qualitative analysis of the observed data, revealing features that are sensitive to the different physical phenomena that affect the rotation of the Moon.

In particular, the analytical theory has found effective application in computer simulating a new type of observation, such as the ILOM , with the purpose to estimate possibilities of the experiment.
One of the important application of the libration theory is the developing the selenocentric coordinate system useful for navigation tasks in the near-moon space. Such kind of the system the Union Selenocentric Reference System was constructed at the university on the basis of absolute coordinates of lunar craters, obtained with simultaneous photographing craters and stars.

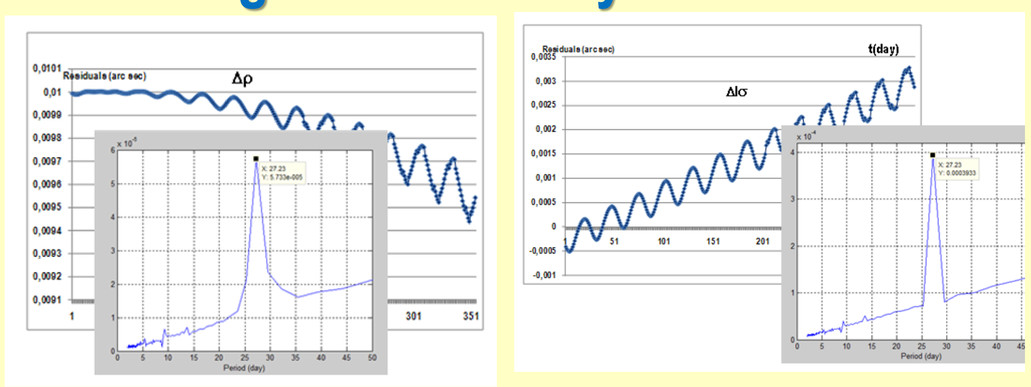
2. Inverse problem of lunar physical libration is formulated and solved. It is shown that selenographic coordinates of polar stars are insensitive to longitudinal librations $\tau(t)$. Comparing coordinates calculated for two models of a rigid and deformable Moon is carried out and components of lunar libration sensitive to Love number k_2 are revealed.

3. Results of the inverse problem: testing accuracy



Error in determining the angles libration in latitude $\Delta\rho$ and in the node $l\Delta\sigma$ does not exceed $\sqrt{2}\varepsilon$, where ε is inaccuracy in the determination of selenographic coordinates *x* and *y*. The simulation confirmed the expected insensitivity to the longitudinal libration $\Delta\tau(t)$ of polar stars coordinates.

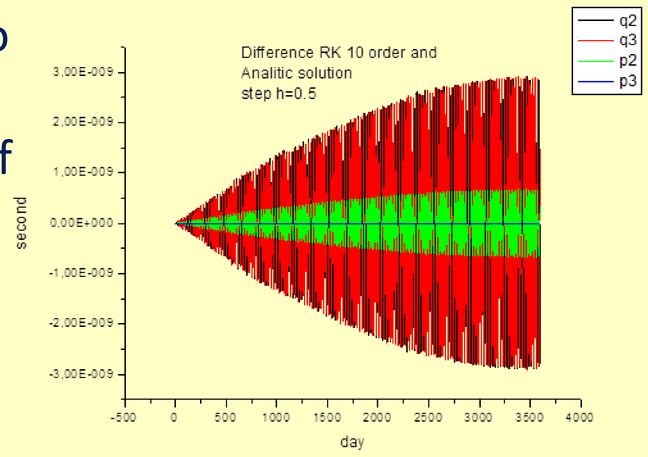
4. Testing the sensitivity to errors in the catalog coordinates of a star



Currently, the inverse problem is used by us to test the effect on the libration parameters of error in the catalogue's coordinates of the observed stars. Preliminary calculations of the effect of errors in stellar coordinates show that the error in 10 milliseconds in the ecliptical coordinates leads to a short-period variations of residual differences with a period of 1 month (27.23 days - the argument F) in 0.05ms $\Delta\rho$ and 0.3 ms in $l\Delta\sigma$.

5. The first stage of numerical approach to construction of the libration theory

Numerical approach can give solution relatively fast, and sometimes it is the only way, to get information about the internal structure of the Moon and external factors affecting its dynamics. To date, we have developed an integrator based on Runge-Kutta method of order 10, which in the solution of the linear problem of physical libration provides accuracy of 10^{-9} seconds in the interval of 10 years when compared with the exact analytical solution.



6. The Union Selenocentric Reference System

Three tasks were addressed in this research: a)the analysis of the mathematical model of the orthogonal coordinate transformation accuracy; b) the identification of the basic dynamic reference system objects with ones that are contained in reducing catalogues; c) the extension of the base points net of the basic dynamic reference system.
The construction of the system were performed using the developed software package "Transformation selenodesic coordinates". During the processing the analysis and investigation of the accuracy of the basic net contained in DSC were carried out:.